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Polar Bi-gyrotropic Single-negative Magnetic Metamaterials: Optical and Magneto-optical Properties

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Abstract

Herein, we report the potential of polar bi-gyrotropic single-negative magnetic metamaterials (that their ε and μ are in the form of non-diagonal tensors) to act simultaneously as reflector and magnetic rotator layer. In other word, we report the development of the single-negative metamaterials at a prominent location in magneto-optics. To understand the optical and magneto-optical properties of polar bi-gyrotropic single-negative layers, we have firstly introduced a transfer matrix method (TMM) based-approach and then we have numerically studied the propagation of electromagnetic wave through such medium. The study of the electromagnetic wave propagation through the polar bi-gyrotropic single-negative material may help to study the unusual behavior of the MO periodic structures (such as magnetophotonic crystals) containing them. The study exposes that such polar bi-gyrotropic single-negative layers can be used to make controlled and to enhance the optical and MO properties of optical.

Keywords: Bi-gyrotropic magnetic medium, Single-negative metamaterial, Polar magnetization, Transfer matrix method.

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1. Introduction

Introduction

Metamaterials are artificial materials and engineered composites in which their electromagnetic properties (ε and μ) can be controlled, because, they make possible the manipulation of fields and waves at subwavelength scales. Advances in simulation and fabrication technologies have brought about broad flexibilities in the design of metamaterials and, consequently, their electromagnetic responses with super performances [1-4]. Single-negative (SNG) materials are two kinds of metamaterials: The ε-negative (ENG) material with $\varepsilon < 0$ while $\mu > 0$, and the μ-negative (MNG) material with $\varepsilon > 0$ while $\mu < 0$. They have significant applications in perfect lens, flat lens imaging and light trapping [5-7].

The SNG materials usually due to including plasmonic or ferromagnetic materials ingle-negative materials have intrinsic optical anisotropy properties [8-10]. On the other hand, many commonly used metamaterial structures exhibit magneto-electric coupling, hence, such media are represented as bi-anisotropic [11-13].

Optical anisotropy of a magnetic material can be seen in the light reflection from material surface and when light passes through the magnetic layers. Among MO effects, magneto-optical polar effect demonstrates itself by variation of light polarization for and longitudinal magnetization when both magnetization and propagation vectors are parallel and perpendicular to the interface [14, 15]. This MO effect may be simultaneously described by nonzero off-diagonal ε and μ tensors, so that the related material is known as a bi-gyrotropic material [15].

In this paper, we have firstly provided a general framework for using the polar bi-gyrotropic single-negative magnetic materials in multilayer structures. The optical and MO properties of such structures are generally calculated by transfer matrix method (TMM). Therefore, a 4×4 TMM approach has been introduced to evaluate such multilayer structures including bi-gyrotropic SNG magnetic materials in polar magnetization configuration. Finally, by using this universal approach, to investigate the potential of reflectance and magneto-optical Kerr rotation performances of polar bi-gyrotropic single-negative magnetic materials, numerical computations have been carried out.

2. Theoretical method

For the purpose of using the potential of bi-gyrotropic SNG materials in multilayer structures, transfer matrix method which can apply bi-gyrotropic SNG materials should be employed. In order to do so, we attempted to generalize the transfer matrix method introduced by Zak et al [16] to bi-gyrotropic SNG materials and propose an approach related to them. The above-mentioned approach and the others introduced by us [17-20] can be applied to structures containing bi-, single- and non-gyrotropic as well as positive and negative materials.

This method is based on definition of two 4×4 matrices: one is boundary matrix and the other propagation matrix related to each layer, which both are directly adopted for applying boundary conditions to the problem. The boundary matrix relates the tangential components of the electric and magnetic fields to the *s* and *p* components of the electric field, while the second matrix (medium propagation matrix), relates the *s* and *p* components of the electric field to the two surfaces of each layer or at any point inside the layers. Total transfer matrix in a multilayer system is obtained by multiplying matrices related to all layers. This matrix presents optical and magneto-optic features of the structure.

To simplify the use of TMM method, components of electric field corresponding to *s* and *p* polarizations are applied. Therefore, other set of fields as $E_s^{(i)}, E_p^{(i)}, E_s^{(r)}$ and $E_p^{(r)}$ are applied; so that *s* and *p* denote components of electric field perpendicular to or parallel with the incident plane, respectively. Moreover, the indices *i* and *r* refer to incident and reflective components of light beam, respectively. Tangential components of electric (E_x and E_y) and magnetic (H_x and H_y) fields are presented in the form of column matrix of *F* and *s* and *p* components of electric field in the form of column matrix of *P*, as follows. Obtaining MO coefficients, we require a matrix relating the components of two matrices of F, and P, which we call it the boundary matrix *A* of the medium.

$$F = A\,P\ ; \qquad F = \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} E_s^{(i)} \\ E_p^{(i)} \\ E_s^{(r)} \\ E_p^{(r)} \end{pmatrix}, \qquad (1)$$

Obtaining the medium boundary matrix for a bi-gyrotropic SNG magnetic medium is the main purpose of the present approach, which can be easily generalized to non-magnetic and single-gyrotropic SNG media. Here, it is attempted to present our considered approach for specific state of polar magnetization in which magnetization vector is oriented perpendicularly to the interface. For a bi-gyrotropic ENG and MNG magnetic medium, electric permittivity tensor (ε) and magnetic permeability (μ) of a medium for polar magnetization configuration are as follows,

$$\overleftrightarrow{\varepsilon}_{ENG,MNG}=\begin{pmatrix}\mp\epsilon_1 & i\epsilon_2 & 0\\ -i\epsilon_2 & \mp\epsilon_1 & 0\\ 0 & 0 & \mp\epsilon_1\end{pmatrix}=\epsilon_1\begin{pmatrix}\mp 1 & iQ_E & 0\\ -iQ_E & \mp 1 & 0\\ 0 & 0 & \mp 1\end{pmatrix}, \quad (2)$$

$$\overleftrightarrow{\mu}_{ENG,MNG}=\begin{pmatrix}\pm\mu_1 & i\mu_2 & 0\\ -i\mu_2 & \pm\mu_1 & 0\\ 0 & 0 & \pm\mu_1\end{pmatrix}=\mu_1\begin{pmatrix}\pm 1 & iQ_M & 0\\ -iQ_M & \pm 1 & 0\\ 0 & 0 & \pm 1\end{pmatrix}, \quad (3)$$

in which, in these equations, $-\epsilon_1$ and μ_1 form the bi-gyrotropic ENG medium, while ε_1 and $-\mu_1$ form the bi-gyrotropic MNG medium. On the other hand, ϵ_2 and μ_2 are magnetic rotation parameters which have a linear relationship with magnetization vector. Furthermore, Q_E and Q_M are electric and magnetic magneto-optic constants, respectively.

To obtain the boundary and propagation matrices, in spite of the difference in details, the reader can follow a similar framework with ref [17,18]. According to the equation $\boldsymbol{D}=\overleftrightarrow{\varepsilon}\,\boldsymbol{E}$ and that in the local coordinate system $D_{z'}=0$, we can derive the relation between *x* and *y'* components of the D-wave for all components of the beams 1, 2, 3, and 4 (where the indices 1 and 2 are dealing with forward waves, while 3 and 4 denote backward waves inside the magnetic bi-gyrotropic medium) in both bi-gyrotropic ENG and MNG medium for specific states of polar magnetization, as follows: for the both bi-gyrotropic ENG and MNG magnetic media:

$$\left(D_{y'}/D_x\right)_{1.2}(ENG)=\left(D_{y'}/D_x\right)_{3.4}(ENG)=\mp i\,, \quad (4)$$

$$\left(D_{y'}/D_x\right)_{1.2}(MNG)=\left(D_{y'}/D_x\right)_{3.4}(MNG)=\pm i\,. \quad (5)$$

In addition, the relations between components of *D* in polar geometry for both bigyrotropic ENG and MNG magnetic media are as below:

$$\left(D_y/D_x\right)_{1.2}(ENG)=-\left(D_y/D_x\right)_{1.2}(MNG)=\mp i\alpha_z^{(1.2)}\,, \quad (6)$$

$$\left(D_z/D_x\right)_{1.2}(ENG)=-\left(D_y/D_x\right)_{1.2}(MNG)=\pm i\alpha_z^{(1.2)}\,. \quad (7)$$

in which $\alpha_z=\cos\theta_2$ and $\alpha_y=\sin\theta_2$, so that θ_2 is the refracted angle in the medium 2 (bi-gyrotropic SNG medium). In addition, similar relations can be acquired for 3 and 4 waves.

By using relations (2)-(7), $\boldsymbol{D}=\overleftrightarrow{\varepsilon}\,\boldsymbol{E}$ and $D_{z'}=0$, the relationship between the components of the electric field for the forward and backward waves in bi-gyrotropic ENG and MNG magnetic media can be obtained as:

$$E_y^{(1.3).(2.4)}(ENG)=-E_y^{(1.3).(2.4)}(MNG)=\left(\mp i\alpha_z^{(1.3).(2.4)}-iQ_E\left(\alpha_y^{(1.2.3.4)}\right)^2\right)E_x^{(1.2.3.4)}, \quad (8)$$

$$E_z^{(1.3).(2.4)}(ENG)=-E_z^{(1.3).(2.4)}(MNG)=\left(\mp i\alpha_z^{(1.3).(2.4)}-iQ_E\left(\alpha_y^{(1.2.3.4)}\right)\left(\alpha_z^{(1.2.3.4)}\right)\right)E_x^{(1.2.3.4)}. \quad (9)$$

The refractive indices of the bi-gyrotropic ENG and MNG magnetic medium to the first order in QE + QM for the forward and backward beams can be obtained from the following equations:

$$n_{(Pol)}^{(1.2)}(ENG)=n_{(Pol)}^{(1.2)}(MNG)=iN\left(1\mp\frac{1}{2}\alpha_z(Q_E+Q_M)\right), \quad (10)$$

$$n_{(Pol)}^{(3.4)}(ENG)=n_{(Pol)}^{(3.4)}(MNG)=iN\left(1\pm\frac{1}{2}\alpha_z(Q_E+Q_M)\right). \quad (11)$$

in which $N=\sqrt{\varepsilon_1\,\mu_1}$.

In the following, we are going to calculate the tangential components of magnetic field (Hx and Hy) in terms of the component of the column matrix *P* by using the relation $\overleftrightarrow{\mu}\,\boldsymbol{H}=\boldsymbol{n}\times\boldsymbol{E}$. In this equation, $\boldsymbol{n}$ is a vector that its magnitude defines the refractive index and its direction denotes the propagation direction. Therefore, the H_x and H_y components of the magnetic field for the whose four waves inside the medium will be as

$$\pm\mu_1 H_x^{(j)}+i\mu_1 Q_M H_y^{(j)}=n^{(j)}\left(\alpha_y^{(j)}E_z^{(j)}-\alpha_z^{(j)}E_y^{(j)}\right), \quad (12)$$

$$\pm i\mu_1 Q_M H_x^{(j)}+\mu_1 H_y^{(j)}=n^{(j)}\alpha_z^{(j)}E_x^{(j)}. \quad (13)$$

in which the upper (lower) signs are related to ENG (MNG) case. Also, it should be noticed that ${\alpha_z}^{(j)}=\cos{\theta_2}^{(j)}$ and ${\alpha_y}^{(j)}=\sin{\theta_2}^{(j)}$ are the cosines and sines of the angles ${\theta_2}^{(j)}$, and *j*=1,2,3,4 according to four waves inside the bigyrotropic ENG and MNG magnetic medium.

By using snell's law and Eqs. (10) and (11), the coefficients of ${\alpha_z}^{(j)}$ and ${\alpha_y}^{(j)}$ for the forward and backward waves in bi-gyrotropic ENG and MNG magnetic media in polar geometry can be obtained as follows:

$${\alpha_y}^{(1.4).(2.3)}=\alpha_y\left(1\pm\frac{1}{2}(Q_E+Q_M)\alpha_z\right), \quad (14)$$

$${\alpha_z}^{(1.2).(3.4)}=\alpha_z\left(\pm 1\mp\frac{1}{2}(Q_E+Q_M)(\alpha_y^2/\alpha_z)\right). \quad (15)$$

By substituting the Eqs. (8)-(11) as well as Eqs. (14) and (15) in Eqs. (12) and (13), the components of magnetic field for four waves inside a bi-gyrotropic ENG and MNG medium in polar cases can be achieved. Hence, up to now, the matrix *F* has been obtained.

However, to derive the boundary matrix *A*, we also have to determine the matrix *P* including *s* and *p* components of electric field inside the bi-gyrotropic ENG and MNG medium. In this way, we can write the *s* and *p* components of electric field for incident and reflected waves inside the bi-gyrotropic SNG magnetic layer as

$${E_s}^{(i.r)}={E_x}^{(1.3)}+{E_x}^{(2.4)}, \quad (16)$$

$${E_p}^{(i.r)}=\left(\boldsymbol{E}^{(1.3)}+\boldsymbol{E}^{(2.4)}\right)\cdot\hat{y}^{(i.r)}, \quad (17)$$

in which unit vectors of $\hat{y}^{(i)}$ and $\hat{y}^{(r)}$ denote the directions of axes $y^{(i)}$ and $y^{(r)}$, respectively. These unit vectors are defined as

$$\hat{y}^{(i.r)}=\pm\alpha_z\hat{y}-\alpha_y\hat{z}\,. \quad (18)$$

Finally, the boundary matrix A for both bi-gyrotropic ENG and MNG materials in polar geometry, according to the relation between tangential components of electric and magnetic fields and *s* and *p* components of electric field, can be obtained as follows: SNG magnetic layer as

$$A_{(ENG)}=\begin{pmatrix}1 & 0 & 1 & 0\\ -\frac{i}{2}{\alpha_y}^2(Q_E-Q_M) & \alpha_z & -\frac{i}{2}\alpha_y^2(Q_E-Q_M) & -\alpha_z\\ \frac{N}{2\mu_1}\alpha_z(Q_E+3Q_M) & \frac{-iN}{\mu_1}(1+\frac{1}{2}NQ_EQ_M) & -\frac{N}{2\mu_1}\alpha_z(Q_E+3Q_M) & \frac{-iN}{\mu_1}(1+\frac{1}{2}NQ_EQ_M)\\ \frac{iN\alpha_z}{\mu_1}(1+\frac{1}{2}Q_EQ_M) & \frac{N}{2\mu_1}(Q_E+3Q_M) & -\frac{iN\alpha_z}{\mu_1}(1+\frac{1}{2}Q_EQ_M) & \frac{N}{2\mu_1}(Q_E+3Q_M)\end{pmatrix}, \quad (19)$$

$$A_{(ENG)}=\begin{pmatrix}1 & 0 & 1 & 0\\ -\frac{i}{2}{\alpha_y}^2(Q_E-Q_M) & \alpha_z & -\frac{i}{2}\alpha_y^2(Q_E-Q_M) & -\alpha_z\\ \frac{N}{2\mu_1}\alpha_z(Q_E+3Q_M) & \frac{-iN}{\mu_1}(1+\frac{1}{2}NQ_EQ_M) & -\frac{N}{2\mu_1}\alpha_z(Q_E+3Q_M) & \frac{-iN}{\mu_1}(1+\frac{1}{2}NQ_EQ_M)\\ \frac{iN\alpha_z}{\mu_1}(1+\frac{1}{2}Q_EQ_M) & \frac{N}{2\mu_1}(Q_E+3Q_M) & -\frac{iN\alpha_z}{\mu_1}(1+\frac{1}{2}Q_EQ_M) & \frac{N}{2\mu_1}(Q_E+3Q_M)\end{pmatrix}. \quad (20)$$

Moreover, the propagation matrix of a bi-gyrotropic ENG and MNG magnetic medium for polar magnetization, by following the formalism of ref [16], is formed as

$$\overline{D}_{POL} = \begin{pmatrix} U\cosh\delta & \pm iU\sinh\delta & 0 & 0 \\ \mp iU\sinh\delta & U\cosh\delta & 0 & 0 \\ 0 & 0 & U^{-1}\cosh\delta & \pm iU^{-1}\sinh\delta \\ 0 & 0 & \mp iU^{-1}\sinh\delta & U^{-1}\cosh\delta \end{pmatrix} \quad (21)$$

in which the upper (lower) sign denotes a bi-gyrotropic ENG (MNG) magnetic medium for polar magnetization; while $U = \exp(kN\alpha_z d)$ and $U = \exp[kNd(Q_E + Q_M)/2]$. In addition, d is of the thickness bi-gyrotropic SNG magnetic medium. Also, k is the wave number. It should be noted that the propagation matrix can be generalized to non-magnetic SNG materials by having the MO constants equal to zero. These matrices are the boundary and propagation matrices for bi-gyrotropic ENG and MNG magnetic media. If we set $Q_E = 0$ or $Q_M = 0$, the related matrices for a single-gyrotropic ENG and MNG magnetic medium can be achieved; and if both QE and QM are set to zero (QE = QM = 0), the matrices A and D for a non-magnetic ENG and MNG medium is obtained.

3. Numerical Simulations and discussion

We are going to investigate the influence of polar magnetization on spectrum responses of the single layer of the bi-gyrotropic ENG and MNG materials and finally multilayer structures (MPCs) which include bi-gyrotropic single-negative magnetic materials.

In our previous work [19] we indicated that the performances of both bi-gyrotropic ENG and MNG layers in presence of longitudinal magnetization were pretty similar together. The results have revealed a nearly perfect reflectance for all values, and a larger Kerr rotation for the bi-gyrotropic ENG layer was obtained rather than MNG one. If we want to compare such situation in presence of polar magnetization, we will find giant Kerr rotations rather than longitudinal magnetization case. In polar geometry, both reflectance and Kerr rotation parameters have larger amounts for a single ENG layer rather than MNG one.

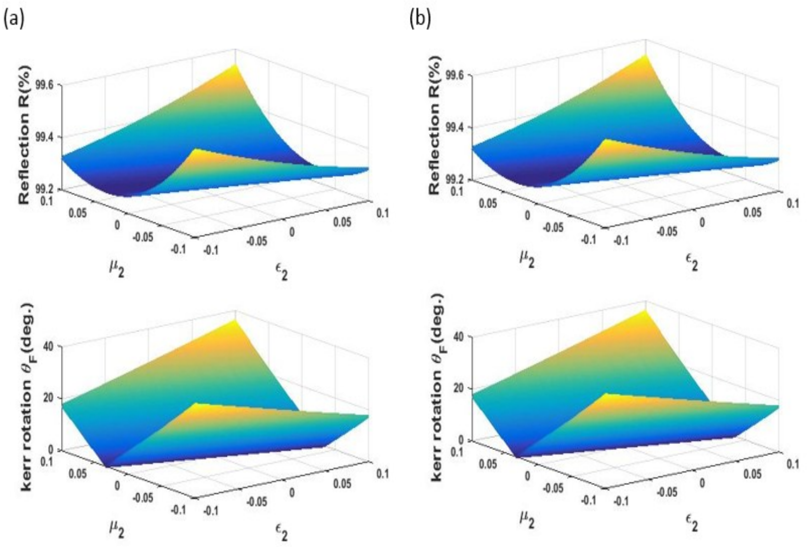

Figure 1: Dependences of reflectance and Kerr rotation of a half-wave bi-gyrotropic (a) ENG layer (with $\epsilon_1 = -3$ and $\mu_1 = 3$) and (b) MNG layer ($\epsilon_1 = 3$ and $\mu_1 = -3$) to the non-diagonal components of ϵ_2 and μ_2, in the case of polar magnetization and for the case of normal incidence of light at wavelength of 1550 nm.

Figures 1 and 2 show the numerical results for both bi-gyrotropic (a) ENG and (b) MNG magnetic media, respectively. In Fig. 1, the reflectance and Kerr rotation of a half-wave bi-gyrotropic (a) ENG layer (with $\epsilon_1 = -3$ and $\mu_1 = 3$) and (b) MNG layer (with $\epsilon_1 = 3$ and $\mu_1 = -3$) as functions of non-diagonal components (ϵ_2 and μ_2) have been illustrated; while Fig. 2 shows such optical and magneto-optical parameters as functions of diagonal components (ϵ_1 and μ_1) for (a) ENG and (b) MNG magnetic media with $\epsilon_2 = \mu_2 = 0.1$ (according to optimal responses in Fig.1). In both figures, the numerical computations have been done for the case of normal incidence of light at wavelength of 1550 nm and in presence of polar magnetization. The results show that, in presence of polar magnetization, the performance of bi-gyrotropic ENG layer is more significant rather than bigyrotropic MNG layer, too.

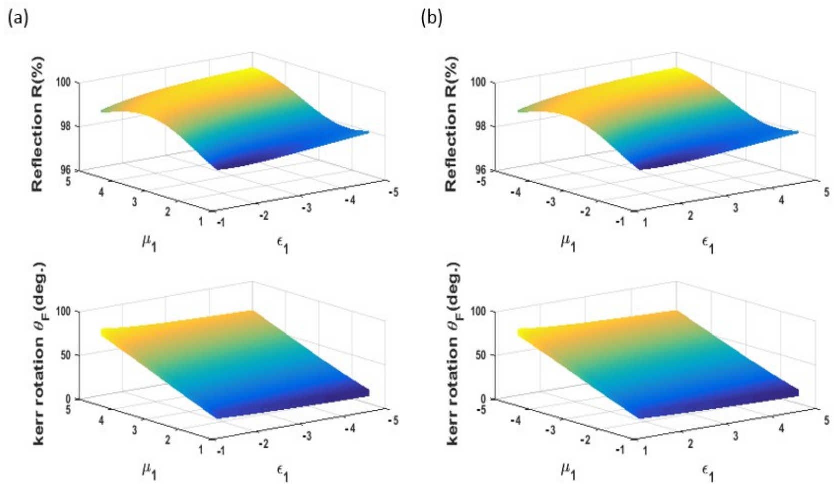

Figure 2: Dependences of reflectance and Kerr rotation of a half-wave bi-gyrotropic (a) ENG layer and (b) MNG layer to the diagonal components of ϵ_1 and μ_1, while the non-diagonal components of $\overleftrightarrow{\varepsilon}$ and $\overleftrightarrow{\mu}$ tensors have been chosed as $\epsilon_2 = \mu_2 = 0.1$, in the case of polar magnetization and for the case of normal incidence of light at wavelength of 1550 nm.

4. Conclusions

The focus of present research has been on the potential of using polar bi-gyrotropic single-negative magnetic materials to act simultaneously as reflector and magneto-optical rotator. Accordingly, we have firstly employed a 4×4 the transfer matrix method to understand the interaction between an electromagnetic field and such magnetic metamaterials. we have numerically demonstrated that a single layer of polar bi-gyrotropic ENG or MNG magnetic material can show notable reflectance and Kerr rotation, simultaneously, and can significantly enhance the operation of multilayer structures.

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