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Self-focusing of hollow Gaussian laser beam in collisionless plasmas and its effect on Stimulated Raman backscattering

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Abstract

In this paper, stimulated Raman backscattering by an intense self-focused hollow Gaussian laser beam (carrying null intensity at the center) in a collisionless plasma has been investigated in the relativistic-ponderomotive regime. The effect of relativistic-ponderomotive self-focusing of an intense hollow Gaussian laser beam (HGLB) on the excitation of electron plasma wave (EPW) and resulting back stimulated Raman scattering have been studied, at relativistic laser power. A paraxial-ray and WKB approximations are used to obtain the coupled equations describing the propagation of the hollow Gaussian laser beam in plasma, excitation of electron plasma wave and SRS back reflectivity. The effects of the order (n) and the pump intensity (a) of HGLB on the focusing/defocusing and intensity of hollow Gaussian beam in plasma, amplitude of the excited electron plasma wave and the back reflectivity of SRS has been explored. It is observed that the parameters n and a play an imperative role in this scheme. The self-focusing of HGLB in plasma is enhances for higher values of n and a, which significantly affected the dynamics of the excitation of EPW and back reflectivity of SRS. The results show that the focusing of waves i. e. pump wave, EPW and scattered wave enhances the back reflectivity of SRS.

Keywords: Hollow Gaussian Laser beam, collisionless plasma, relativistic-ponderomotive nonlinearity, electron plasma wave, stimulated Raman backscattering

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1. Introduction

The propagation of an ultra-intense short duration laser pulses through plasmas has been a subject of significant research interest due to its relation to many potential applications such as fast ignition in inertial confinement fusion (ICF), compact laser-driven accelerators and new radiation sources [1-6]. The overall efficiency of these applications depends on the propagation of intense laser beams to longer distance (several Rayleigh lengths) in the plasmas without loss of the energy. The propagation of an intense laser pulses in plasma encounters various parametric instabilities such as self-focusing, filamentation, stimulated Raman scattering (SRS) from electron plasma wave, stimulated Brillouin scattering (SBS) from ion acoustic wave, two-plasmon instability etc [7-11]. These instabilities create a hindrance in laser plasma coupling i. e. energy of laser pulse is not efficiently coupled with plasmas. Due to redistribution of laser energy inside the plasmas, the overall efficiency of the laser beam energy transport over long distances in plasmas would be strongly limited. Therefore, the study of these instabilities at very high laser power flux is very important to the success of above applications.

Self-focusing and stimulated Raman backscattering (SRBS) of relativistic laser pulse in plasmas are most promising topics of investigation in laser plasma interaction [12-15]. Self-focusing plays an important role in the guidance of laser beams in plasmas and essentially initiated due to increase of the on-axis index of refraction relative to the edges of the wave front of the laser beam, while SRS instability of intense laser pulses in plasma occurs when the incident laser light resonantly couples with EPW and a backscattered light wave. These two processes have been observed in laser-driven fusion experiments (direct and indirect inertial confinement fusion) especially in high density plasma. Self- focusing of an intense laser beam in plasma produces energetic electrons, which may preheat the fusion fuel and reduce the compression rate [16], while SRS instability is responsible to the possible large conversion of laser energy into scattered light. This instability have a maximum growth rate for the scattered wave propagating in the backward direction, which reduce the efficiency and symmetry of the energy transfer from the laser to the target [17]. The back reflectivity of SRS is significantly affected in the presence of self-focused laser beam and EPW.

A lot of theoretical and experimental work has been reported on the self-focusing of intense laser beam in plasma and its effect on the generation of EPW and SRBS process under different time scale of laser pulse in the past [18-30]. The extent of self-focusing of laser beam and back reflectivity of SRS in plasma depends on nonlinearities associated with plasmas and the spatial profile of laser beam. Many earlier as well as current studies of self-focusing of laser beam in plasma have been carried out under the assumption of uniform laser beams having Gaussian intensity distribution (TEM_{00} mode) along with ponderomotive or relativistic nonlinearity [31-35]. However, the behavior of laser beams is different for different intensity profile in plasmas. Only a few investigations have been reported on SRBS by diversified laser beam profiles such as elliptical Gaussian beam [36] and rippled Gaussian beam [37]. It has been observed that the past experimental results on SRS process are not matched with the theoretical results. This is because the pump laser beams is not perfectly smooth in many experimental situations but are the superposition of higher order modes. For the better understanding of SRS process at higher laser intensities, the role of higher order modes must be included in the theoretical analysis. Particularly, the hollow Gaussian intensity profile of laser beam [38-42] that can be expressed as the superposition of a series of Laguerre-Gaussian modes and considered as an optical beam with null intensity at the center is a best example of TEM^{*}_{01} mode. Such beams can be used to guide, focus and trap neutral atoms and having a wide range of applications in many fields such as plasma physics, atomic and modern optics, atmospheric science and bio-photonics. Similarly, nonlinearities associated with laser plasma interaction play important role in self focusing of laser beam and SRS process. When an ultra intense laser beam propagates through collisionless plasma, the dielectric constant/refractive index of plasma is modified by combined effect of the ponderomotive and relativistic nonlinearities, which leads to the self focusing of the laser beam [12]. Relativistic self-focusing is caused by the mass increase in electrons travelling at speed approaching the speed of light, which increases the refractive index by decreasing the plasma frequency at higher intensity. On the other hand, ponderomotive nonlinearity is set up by the relativistic-ponderomotive force, which pushes electrons away from the region where the laser beam is more intense and decreases the electron density. Consequently, the refractive index becomes higher at the higher intensity region and self–focusing becomes enhance due to relativistic mechanism. These nonlinearities are operative at different time scales according to the inequalities: i) $\tau < \tau_{pe}$ (when only relativistic nonlinearity is operative), ii) $\tau_{pe} < \tau < \tau_{pi}$ (when both i. e. relativistic and ponderomotive nonlinearities are present), where τ is the laser pulse duration, τ_{pe} is the electron plasma period, and τ_{pi} is the ion plasma period. Since relativistic nonlinearity arises instantaneously in the system, so it requires relatively higher threshold laser power than the ponderomotive nonlinearity. Ponderomotive nonlinearity modifies the intensity distribution of the laser beam and it is dominant when the characteristic time of observation is greater than the diffusion time of the charge carriers. The literature review reveals that the study of SRBS instability in plasma by hollow Gaussian laser beam have not been investigated under relativistic-ponderomotive regime; except that the recent study of SRBS by relativistic/ponderomotive self-focusing of hollow Gaussian laser beam in plasma [43, 44].

This paper presents stimulated Raman backscattering of an intense hollow Gaussian laser beam (for different orders) in collisionless plasma along with the combined effect of relativistic and ponderomotive nonlinearities. In this process, pump laser beam (ω_0, k_0) interacts with pre excited electron plasma wave (ω_e, k_e) and generates stimulated Raman scattered wave of frequency ($\omega_0 - \omega_e$) and wave number ($k_0 - k_e$). The paraxial-ray approximation [45, 46] has been used to formulate the analytical model. This paper is arranged as follows: In section 2, an expressions for the nonlinear dielectric function of the plasma and the propagation of an intense hollow Gaussian laser beam in collisionless plasma under the combined effect of relativistic-ponderomotive nonlinearity is given. The excitation of EPW is studied in section 3. The expressions for the beamwidth parameter of scattered beam and backreflectivity of SRS are given in section 4. In Section 5, we have presented a detailed discussion of numerical results carried out for the relevant parameter. The main conclusions are drawn in the last section.

2. Analytical Model

The electron density distribution of plasma is modified by the relativistic-ponderomotive force of the laser beam while it propagates through a collisionless plasma. Therefore, the dielectric constant of plasma would be changed which leads to a modification in the propagation characteristics of laser beam in plasma. The effect of relativistic-ponderomotive force on the modified electron density distribution and the propagation of an intense HGLB in plasma are given in subsections 2.1 and 2.2.

2.1 Nonlinear Dielectric constant of the Plasma

Consider the propagation of an intense hollow Gaussian beam (HGB) of angular frequency ω_0 and wave vector k_0 in a collisionless plasma along the z axis. The initial field distribution of hollow Gaussian laser beam can be defined as

$$(E_0)_{z=0} = E_{00}\left(\frac{r^2}{2r_0^2}\right)^n \exp\left(-\frac{r^2}{2r_0^2}\right) \tag{1}$$

where r_0 is the initial beam width of the beam, n is the order of the HGB and is a positive integer and E_{00} is the maximum amplitude of the HGB obtained at $r = r_{\max} = r_0\sqrt{2n}$. Eq. (1) represents a fundamental Gaussian laser beam at $n = 0$. For HGB, transforming the (r, z) coordinate to the (η, z) coordinate by the relation [41]

$$\eta = \frac{r}{r_0 f_0(z)} - \sqrt{2n} \tag{2}$$

where η is a reduced radial coordinate, f_0 is the dimensionless beam width parameter and $r = r_0 f_0\sqrt{2n}$ at $\eta = 0$ is the position of the maximum irradiance for the propagating beam. However, the position of maximum intensity lies at $r = 0$ in the case of Gaussian profile of laser beam.

The dielectric constant of the plasma is given by

$$\varepsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} \tag{2}$$

where $\omega_{p0} = 4\pi n_0 e^2 / m_0$ is the plasma frequency, n_0 is the density of plasma electrons in the absence of laser beam , m_0 is the mass of the electron, and e is the electronic charge. The dielectric constant of the plasma is modified by relativistic and ponderomotive nonlinearities, because of the relativistic change in the mass of electron and the modification of the background electron density due to ponderomotive nonlinearity. The relativistic-ponderomotive force on electron in the presence of the laser field is given by [47, 48]

$$F_P = -m_0 c^2 \nabla(\gamma - 1) \tag{3}$$

where γ is the relativistic Lorentz factor and is given by

$$\gamma = \left[1 + \frac{e^2}{c^2 m_0^2 \omega_0^2} E.E^*\right]^{\frac{1}{2}}$$

The modified electron density (n_e) due to relativistic-ponderomotive force is given as [47]

$$n_e = n_0 + n_2 = n_0 + \frac{c^2 n_0}{\omega_{p0}^2}\left(\nabla^2\gamma - \frac{(\nabla\gamma)^2}{\gamma}\right) \tag{4}$$

where n_2 is the nonlinear variation of the density together with the relativistic correction i.e. modified electron density due to the ponderomotive force. The modified electron density is given by

$$\frac{n_e}{n_0} = 1 + \frac{c^2}{\omega_{p0}^2}\left\{ \frac{a}{r_0^2 f_0^4 2^{2n}\gamma}(\eta+\sqrt{2n})^{4n}\exp[-(\eta+\sqrt{2n})^2]\left[\frac{8n^2}{(\eta+\sqrt{2n})^2} + 2(\eta+\sqrt{2n})^2 - 8n - 2\right] \right.$$
$$\left. - \frac{a^2}{r_0^2 f_0^6 2^{2n}\gamma^3}(\eta+\sqrt{2n})^{8n}\exp[-2(\eta+\sqrt{2n})^2]\left[\frac{4n^2}{(\eta+\sqrt{2n})^2} + (\eta+\sqrt{2n})^2 - 4n\right]\right\} \tag{5}$$

Now, the effective dielectric constant of the plasma in the presence of relativistic-ponderomotive nonlinearity can be expressed as

$$\varepsilon(r,z) = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left[\frac{n_e}{n_0\gamma}\right] \tag{6}$$

For hollow Gaussian laser beam, the dielectric function $\varepsilon(\eta, z)$ around the maximum ($\eta = 0$) under the paraxial-ray approximation can be expressed as [49]

$$\varepsilon(\eta,z) = \varepsilon_0(z) - \eta^2\varepsilon_2(z) \tag{7}$$

where $\varepsilon_0(z)$ and $\varepsilon_2(z)$ are the coefficients associated with η^0 and η^2 in the expansion of $\varepsilon(\eta,z)$ around $\eta=0$. The dielectric functions $\varepsilon_0(z)$ and $\varepsilon_2(z)$ are obtained by expanding the dielectric function $\varepsilon(\eta, z)$ in the paraxial regime around the position of maximum intensity as

$$\varepsilon_0(z) = \varepsilon(\eta,z)_{\eta=0} = 1 - \frac{\omega_{p0}^2}{\omega_0^2}\left(\frac{1}{(1+g_0)^{1/2}}\right) + \frac{1}{\rho_0^2 f_0^2}\left(\frac{2g_0}{(1+g_0)}\right) \tag{8}$$

and

$$\varepsilon_2(z) = -\left(\frac{\partial\varepsilon(\eta,z)}{\partial\eta^2}\right)_{\eta=0} = \frac{\omega_{p0}^2}{\omega_0^2}\left(\frac{1}{(1+g_0)^{3/2}}\right)g_0 - \frac{1}{\rho_0^2 f_0^2}\left(\frac{4g_0^2}{(1+g_0)^2}\right) \tag{9}$$

where $g_0 = \frac{a}{f_0^2} n^{2n}\exp(-2n)$ and a (= αE_{00}^2) is the initial intensity of laser beam.

2.2 Propagation of hollow Gaussian laser beam in collisionless plasma

The propagation of the hollow Gaussian laser beam in collisionless plasma is governed by the wave equation

$$\nabla^2 E_0 + \frac{\omega_0^2}{c^2}\varepsilon_0(r,z)E_0 = 0 \tag{10}$$

The solution of Eq. (10) can be written as [45]

$$E_0(r,z) = \hat{i}\, A(r,z)\exp\left(-i\int k_0(z)dz\right) \tag{11}$$

where $A(r, z)$ is a complex amplitude of the wave, and $k_0(z) = \frac{\omega_0}{c}\sqrt{\varepsilon_0(z)}$ and and $\varepsilon_0(z)$ is the dielectric function, corresponding to the maximum electric field on the wavefront of the HGB. Substituting $E_0(r, z)$ from Eq. (11) in Eq. (10) and neglecting the term $\left(\partial^2 A/\partial z^2\right)$, one obtain

$$2ik\frac{\partial A}{\partial z} + iA\frac{\partial k}{\partial z} = \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega_0^2}{c^2}(\varepsilon - \varepsilon_0)A \tag{12}$$

The complex amplitude $A(r, z)$ may be defined as

$$A(r,z) = A_0(r,z)\exp\left(-ik_0(z)S(r,z)\right) \tag{13}$$

where $S(r, z)$ is the eikonal associated with the HGB with A_0 and S are the real parameters. Substituting $A(r, z)$ from Eq. (13) in Eq. (12) and separating the real and imaginary parts, one can obtain

$$\frac{2S}{k_0}\frac{\partial k_0}{\partial z} + 2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k_0^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) + \frac{\omega_0^2}{k_0^2 c^2}(\varepsilon - \varepsilon_0) \tag{14}$$

and

$$\frac{\partial A_0^2}{\partial z} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + \frac{A_0^2}{k_0}\frac{\partial k_0}{\partial z} \tag{15}$$

With the help of Eq. (2), Eqs.(14) and (15) in terms of variables (η, z) can be expressed as,

$$\frac{2S}{k_0}\frac{\partial k_0}{\partial z} + 2\frac{\partial S}{\partial z} + \frac{1}{r_0^2 f_0^2}\left(\frac{\partial S}{\partial \eta}\right)^2 = \frac{1}{k_0^2 r_0^2 f_0^2 A_0}\left(\frac{\partial^2 A_0}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial A_0}{\partial \eta}\right) + \frac{\omega_0^2}{k_0^2 c^2}(\varepsilon - \varepsilon_0) \tag{16}$$

and

$$\frac{\partial A_0^2}{\partial \eta} + \frac{A_0^2}{r_0^2 f_0^2}\left(\frac{\partial^2 S}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S}{\partial \eta}\right) + \frac{1}{r_0^2 f_0^2}\frac{\partial A_0^2}{\partial \eta}\frac{\partial S}{\partial \eta} + \frac{A_0^2}{k_0}\frac{\partial k_0}{\partial z} \tag{17}$$

For $\eta << \sqrt{2}n$, the amplitude A_0 is defined as [45, 46]

$$A_0^2 = \frac{E_0^2}{2^{2n} f_0^2}(\sqrt{2n}+\eta)^{4n}\exp[-(\sqrt{2n}+\eta)^2] \tag{18}$$

and the eikonal of the pump beam is given by

$$S(\eta,z) = \frac{(\sqrt{2n}+n)^2}{2} r_0^2 f_0 \frac{df}{dz} + \varphi(z) \tag{19}$$

where $\varphi(z)$ is a function of z and $f_0(z)$ is the beam width parameter for the HGB. Substituting Eqs. (18) and (19) into Eq. (16), equating the coefficients of η^0 and η^2 on both sides of the resulting equation, and using the boundary conditions $f_0|_{z=0} = 1$ and $\left.\frac{df_0}{dz}\right|_{z=0} = 0$, one obtains

$$\varepsilon_0 f_0 \frac{d^2 f_0}{d\xi^2} = \left(\frac{4}{f_0^2} - \rho_0^2 \varepsilon_2\right) \tag{20}$$

where $\xi = \left(c/r_0^2\omega_0\right)z$ is the dimensionless distance of propagation and $\rho_0 = \left(r_0\omega_0/c\right)$ is the dimensionless initial beam width. Equation (20) describes the beam width of hollow Gaussian laser beam with the distance of propagation in collisionless plasma, when both relativistic and ponderomotive nonlinearities are simultaneously operatives.

3. Excitation of electron plasma wave

Due to ponderomotive force and the relativistic effects, the background electron density of plasma is modified. Therefore, the refractive index increases and the laser beam get focused in the plasma. The intensity of laser beam becomes very high at the positions where laser beam is focused in plasma. The amplitude of EPW, which depends upon the background electron density gets strongly coupled to the laser beam and leads to its excitation. In order to investigate the effect of this coupling on the excitation of EPW in the presence of relativistic and ponderomotive nonlinearities, we have set up the equation of the electron plasma wave by using the fluid equations viz. equation of continuity, the equation of motion and Poisson's equation. The electron density variation in the electron plasma wave (neglecting the contribution of the ions) is given by [34]

$$\frac{\partial^2 N_e}{\partial t^2} + 2\Gamma_e\frac{\partial N_e}{\partial t} - v_{th}^2\nabla^2 N_e + \frac{\omega_{p0}^2}{\gamma}\left(\frac{n_e}{n_0}\right)N_e = 0 \tag{21}$$

where $v_{th}^2 = \gamma_e k_b T_0/m_0$ is the electron thermal velocity, γ_e is the ratio of specific heat and Γ_e is the Landau damping factor [10]. Using the WKB and paraxial ray approximations [45,46], the solution of Eq. (21) can be expressed as

$$N_e = N_{e0}(r,z)\exp[i(\omega_e t - k_e z)] \tag{22}$$

where N_{e0} is the complex function for r and z, ω_e and k_e are the frequency and the propagation vector of the electron plasma wave related by the following dispersion relation:

$$\omega_e^2 = \frac{\omega_{p0}^2}{\gamma}\frac{n_e}{n_0} + k^2 v_{th}^2 \quad (23)$$

Using Eq. (23) and substituting Eq. (22) into (21), we get

$$-\omega_e^2 N_{e0} + 2\omega_e i\Gamma_e N_{e0} - v_{th}^2\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)N_{e0} + 2ik_e v_{th}^2 \frac{\partial N_{e0}}{\partial z} + k_e^2 v_{th}^2 N_{e0} + \frac{\omega_{p0}^2}{\gamma}\left(\frac{n_e}{n_0}\right)N_{e0} = 0 \quad (24)$$

Further, we express N_{e0} as

$$N_{e0} = N_{e00}\exp(-ik_e S_e(r,z)) \quad (25)$$

where N_{e00} is the real functions for r and z and S_e is the eikonal. Substituting for N_{e0} from Eq. (25) into Eq. (24), taking $\Gamma_e \cong 0$, transforming the (r, z) coordinate in to (η, z) coordinate by using Eq. (2) and separating the real and imaginary parts, we obtain

$$2\frac{\partial S_e}{\partial z} + \frac{1}{r_0^2 f_0^2}\left(\frac{\partial S_e}{\partial \eta}\right)^2 = \frac{1}{k_e^2 r_0^2 f_0^2 N_{e00}} \times \left(\frac{\partial^2 N_{e00}}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial N_{e00}}{\partial \eta}\right) + \frac{\omega_{p0}^2}{k^2 v_{th}^2}\left(1 - \frac{n_e}{n_0 \gamma}\right) \quad (26)$$

and

$$\frac{\partial N_{e00}^2}{\partial z} + \frac{1}{r_0^2 f_0^2}\frac{\partial S_e}{\partial \eta}\left(\frac{\partial N_{e00}^2}{\partial \eta}\right) + \frac{N_{e00}^2}{r_0^2 f_0^2}\left(\frac{\partial^2 S_e}{\partial \eta^2} + \frac{1}{(\sqrt{2n}+\eta)}\frac{\partial S_e}{\partial \eta}\right) + \frac{2\Gamma_e \omega_e N_{e00}^2}{k_e v_{th}^2} = 0 \quad (27)$$

For the paraxial ray approximation, i.e. for $\eta \ll \sqrt{2n}$, the solution of Eqs. (26) and (27) can be written as

$$N_{e00}\big|_{z=0} = \frac{n_{e00}}{f_e 2^n}(\eta + \sqrt{2n})^{2n}\left(\frac{r_0 f_0}{a_e f_e}\right)^{2n} \times \exp\left(-\frac{r_0^2 f_0^2}{a_0^2 f_p^2}(\eta + \sqrt{2n})^2 - 2k_d z\right) \quad (28)$$

where $k_d = \dfrac{\Gamma_e \omega_e}{k_e^2 v_{th}^2}$ is the damping factor, n_{e00} is the initial density associated with the EPW, a_e and f_e are radius the dimensionless beam width parameter of EPW. The eikonal of the EPW is given by

$$S_e = (\eta + \sqrt{2n})^{2n}\frac{r_0^2 f_0^2}{2 f_e}\frac{\partial f_e}{\partial z} + \phi_e(z) \quad (29)$$

where ϕ_e is a z dependent variable that can be considered as a constant. Now using Eqs. (28) and (29) in Eq. (26) and equating the coefficients of η^2 on both sides under the boundary conditions at $z = 0$, $f_e = 1$ and $df_e/dz = 0$, we obtain

$$\frac{\partial^2 f_e}{\partial \xi^2} = \left(\frac{f_e \rho_0^2}{f_0^2}\right)\left[\frac{1}{k_e^2 r_0^2 f_0^2}\left(\frac{r_0 f_0}{a_e f_e}\right)^4 - \frac{\omega_{p0}^2}{k_e^2 v_{th}^2}\left(\frac{g}{(1+g)^{3/2}} - \frac{c^2}{\omega_{p0}^2 r_0^2 f_0^2}\left(\frac{4g^2}{(1+g)^2}\right)\right)\right] \quad (30)$$

Eq. (30) describes the variation in the dimensionless beamwidth parameter of EPW with the distance of propagation in the collisionless plasma.

4. Stimulated Raman backscattering

In laser plasma interaction, stimulated Raman backscattering is a parametric process in which a laser beam is backscattered off the electron plasma density fluctuations i. e. an incident laser pulse is scattered by the electron-density perturbations of a plasma wave. In order to formulate the expression for back reflectivity of SRS, consider the high frequency electric field E_H, which is the sum of electric field of scattered wave E_S and electric field of pump laser beam E_0

$$E_H = E_0\exp(i\omega_0 t) + E_S\exp(i\omega_S t) \quad (31)$$

where ω_0 and ω_S represents the frequency of the incident laser beam and scattered wave frequency respectively. The electric field E_H satisfies the wave equation

$$\nabla^2 E_H - \nabla(\nabla\cdot E_H) = \frac{1}{c^2}\frac{\partial^2 E_H}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial J_H}{\partial t} \quad (32)$$

where J_H is the total current density vector in the presence of the high frequency electric field E_T. Equating the terms at frequency ω_S, we get the wave equation for scattered field

$$\nabla^2 E_S + \frac{\omega_S^2}{c^2}\left[1-\frac{\omega_{p0}^2}{\omega_S^2}\frac{n_e}{n_0\gamma}\right]E_S = \frac{1}{2}\frac{\omega_{p0}^2}{c^2}\frac{\omega_S}{\omega_o}\frac{n^*}{n_o}E_O \tag{33}$$

The solution of E_S can be written as

$$E_S = E_{S0}(r,z)\exp(ik_{S0}z) + E_{S1}(r,z)\exp(-ik_{S1}z) \tag{34}$$

where E_{S1} and E_{S0} are slowly varying function of r and z, k_{S0} and k_{S1} are the propagation constant of scattered wave, and

$$k_{s0}^2 = \frac{\omega_S^2}{c^2}\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\right) = \frac{\omega_S^2}{c^2}\varepsilon_{S0} \tag{35}$$

k_{S1} and ω_S satisfy phase matching conditions [50] i. e.

$\omega_S = \omega_0 - \omega_e$ and $k_{s1} = k_0 - k_e$.

Using Eq. (34) in (33), we obtain

$$-k_{S0}^2 E_{S0} + 2ik_{S0}\frac{\partial E_{S0}}{\partial z} + \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)E_{S0} + \frac{\omega_S^2}{c^2}\varepsilon_S(r,z)E_{S0} = 0 \tag{36}$$

$$-k_{S1}^2 E_{S1} - 2ik_{S1}\frac{\partial E_{S1}}{\partial z} + \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)E_{S1} + \frac{\omega_S^2}{c^2}\varepsilon_S(r,z)E_{S1} = \frac{1}{2}\frac{\omega_{p0}^2}{c^2}\frac{\omega_S}{\omega_0}\frac{n^*}{n_0}E_0 e^{-ik_0S_0} \tag{37}$$

where $\varepsilon_S(r,z) = \varepsilon_{S0} + \frac{\omega_{P0}^2}{\omega_S^2}\left(1-\frac{n_e}{n_0\gamma}\right)$

The solution of Eq. (37) can be written as

$$E_{S1} = E_{S1}^{'}\exp(-ik_0S_0) \tag{38}$$

Substituting this into Eq. (36) one obtain the following equation

$$E_{S1}^{'} \cong -\frac{1}{2}\left(\frac{\omega_{p0}^2}{c^2}\right)\frac{n^*}{n_0}\left(\frac{\omega_S}{\omega_0}\right)\frac{\hat{E}E_0}{\left[k_{S1}^2 - k_{S0}^2 - \frac{\omega_{p0}^2}{c^2}\left(1-\frac{n_e}{n_0\gamma}\right)\right]} \tag{39}$$

where $\hat{E}$ is the unit vector along E.

The solution of Eq. (36) may be written as

$$E_{S0} = E_{S00}\exp(ik_{S0}S_c) \tag{40}$$

Substituting Eq. (40) into Eq. (36) and separating the real and imaginary parts, we obtain the following equations:

$$2\frac{\partial S_c}{\partial z} + \left(\frac{\partial S_c}{\partial r}\right)^2 = \frac{1}{k_{S0}^2 E_{S00}}\left(\frac{\partial^2 E_{S00}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{S00}}{\partial r}\right) + \frac{\omega_{p0}^2}{\varepsilon_{S0}c^2}\left(1-\frac{n_e}{n_0\gamma}\right) \tag{41}$$

and

$$\frac{\partial E_{S00}^2}{\partial z} + \frac{\partial S_C}{\partial r}\frac{\partial E_{S00}^2}{\partial r} + E_{S00}^2\left(\frac{\partial^2 S_C}{\partial r^2} + \frac{1}{r}\frac{\partial S_C}{\partial r}\right) = 0 \tag{42}$$

Transform the (r,z) coordinate into the (η,z) coordinate by using the Eq. (2) and the solution of the Eqs. (41) and (42) can be written as

$$E_{S00}^2 = \frac{B_S^2}{2^{2n}f_S^2}\left(\sqrt{2n}+\eta\right)^{4n}\left(\frac{r_0 f_0}{b_S f_S}\right)^{4n} \times \exp\left(-\frac{r_0^2 f_0^2}{b_S^2 f_S^2}\left(\sqrt{2n}+\eta\right)^2\right) \tag{43}$$

and

$$S_c = \frac{(\sqrt{2n}+\eta)^2}{2}\frac{r_0^2 f_0^2}{f_S}\frac{\partial f_S}{\partial z} + \phi_S(z) \tag{44}$$

where B_S is the amplitude of the scattered beam, whose value is to be determined later by applying boundary condition, b_S is the initial beam width of the scattered wave and f_S the dimensionless beam width parameter of the scattered beam. Using Eq.(43) and (44) into (41) and equating the coefficients of η^2, we obtain the dimensionless beam width parameter of the scattered beam (f_S)

$$\frac{d^2 f_S}{d\xi^2} = \left(\frac{\rho_0^2}{f_0^2} f_S\right)\left[\begin{matrix}\frac{1}{k_{S0}^2}\left(\frac{r_0 f_0}{b_S f_S}\right)^4 + \\ \frac{\omega_S^2}{k_{S0}^2 c^2}\left(\frac{\omega_{p0}^2 g}{2\omega_0^2 (1+g)^{3/2}} - \frac{1}{\rho_0^2 f_0^2}\frac{4g^2}{(1+g)^2}\right)\end{matrix}\right] \quad (45)$$

where $f_S = 1,\ df_S / dz = 0$ at $z = 0$.

The value of B_S may be obtained on applying appropriate boundary conditions

$$E_S = E_{S0}(r,z)e^{ik_{S0}Z} + E_{S1}(r,z)e^{-ik_{S1}Z} = 0 \quad \text{at } z = z_c \quad (46)$$

where z_c (= $L - z$, L is the interaction length) is the distance at which amplitude of the scattered wave is zero. z_c is chosen sufficiently large so that N_{e00} is nearly zero. Therefore at $z = z_c$, one can obtain

$$B_s = \frac{\frac{1}{2^{n+1}}\left(\frac{\omega_{p0}^2}{c^2}\right)\frac{n_{e00}}{n_0}\left(\frac{\omega_S}{\omega_0}\right)}{k_{S1}^2 - k_{S0}^2 - \frac{\omega_p^2}{c^2}\left(1 - \frac{n_e}{n_0\gamma}\right)} \times \frac{E_{00} f_S(z_c)}{f_e(z_c) f_0(z_c)}\left(\frac{r}{a f_e(z_c)}\right)^{2n} \times \left(\frac{b_S f_S(z_c)}{r}\right)^{2n}\left(\frac{r}{r_0 f_0(z_c)}\right)^{2n}\frac{e^{-i(k_{S0}S_c + k_o S_0)}}{e^{i(k_{S1}z_c + k_{S0}z_c)}} \quad (47)$$

with the condition

$$\frac{1}{b_S^2 f_S^2(z_c)} = \frac{1}{r_0^2 f_0^2(z_c)} + \frac{1}{a_e^2 f_e^2(z_c)} \quad (48)$$

where f_0 (z_c), $f_e(z_c)$ and $f_S(z_c)$ are the values of dimensionless beam width parameters of pump laser beam (HGB), electron plasma beam and scattered beam at $z = z_c$.

By using Eqs. (34), (38), (39), (43), (44) and (47), we get expression for backreflectivity B_R (i. e. ratio of the scattered wave intensity to the input pump wave intensity) of SRS as

$$B_R = \frac{|E_S|^2}{|E_0|^2}$$

$$B_R = \frac{1}{2^{4n+2}}\left(\frac{\omega_{p0}^2}{c^2}\right)^2\left(\frac{\omega_S}{\omega_0}\right)^2\left(\frac{n_{e00}}{n_0}\right)^2 \times \frac{(\eta + \sqrt{2n})^{8n}}{\left[k_{S1}^2 - k_{S0}^2 - \omega_{p0}^2\frac{n_e}{n_0\gamma}\right]^2} \times \left[I_1 + I_2 - I_3\right] \quad (49)$$

where

$$I_1 = \frac{f_S^2(z_c)}{f_0^2(z_c) f_e^2(z_c)}\frac{1}{f_S^2}\left(\frac{f_0}{f_0(z_c)}\right)^{4n}\left(\frac{f_S}{f_S(z_c)}\right)^{4n} \times \left(\frac{r_0 f_0}{a_e f_e(z_c)}\right)^{4n}\exp\left[-2k_d z_c - \frac{r_0^2 f_0^2}{b_S^2 f_S^2}(\eta + \sqrt{2n})^2\right]$$

$$I_2 = \frac{1}{f_0^2 f_e^2}\left(\frac{r_0 f_0}{a_e f_e}\right)^{4n} \times \exp\left[-2k_d z - (\eta + \sqrt{2n})^2 - \frac{r_0^2 f_0^2}{a_e^2 f_e^2}(\eta + \sqrt{2n})^2\right]$$

$$I_3 = \frac{f_S(z_c)}{f_0(z_c) f_e(z_c)}\frac{1}{f_0 f_e f_S}\left(\frac{f_0}{f_0(z_c)}\right)^{2n} \times \left(\frac{f_S}{f_S(z_c)}\right)^{2n}\left(\frac{r_0 f_0}{a_e f_e(z_c)}\right)^{2n}\left(\frac{r_0 f_0}{a_e f_e}\right)^{2n}\exp[-k_d(z + z_c)] \times \exp\left[-(\eta + \sqrt{2n})^2 - \frac{r_0^2 f_0^2}{a_e^2 f_e^2}(\eta + \sqrt{2n})^2 - \frac{r_0^2 f_0^2}{b_S^2 f_S^2}(\eta + \sqrt{2n})^2\right] \times \cos(k_{S1} + k_{S0})(z - z_c)$$

Equation (49) gives the expression for back reflectivity of SRS by hollow Gaussian laser beam in plasma with relativistic-ponderomotive nonlinearity.

5. Numerical results and discussion

In order to analyze the effect of laser intensity (a) and order of HGLB (n) on the self focusing of hollow Gaussian laser beam in plasma, excitation of EPW and back reflectivity of SRS, equations (18), (20), (28), (45) and (49) have been numerically solved for following set of parameters:

$\omega_0 = 1.778 \times 10^{15}$ rad/sec, $r_0 = 20\mu$m, $\omega_{p0} = 0.3\omega_0$, $a_e = 10\mu$m, $v_{th} = 0.1c$, $n = 1$, 2 and 3.

Laser intensities corresponding to $a = 1$, 1.5 and 2 are 1.21×10^{18}W/cm^2, 2.72×10^{18}W/cm^2 and 4.84×10^{18}W/cm^2 respectively. The boundary conditions for incident laser beam (pump wave), electron plasma wave and scattered wave are:

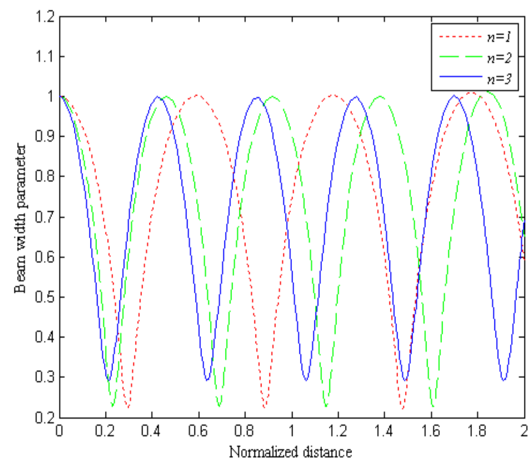
$$\frac{df_0}{dz} = 0, f_0 = 1 \text{ at } z = 0 \quad , \quad \frac{df_e}{dz} = 0, f_e = 1 \text{ at } z = 0$$

and $$\frac{df_s}{dz} = 0, f_S = 1 \text{ at } z = 0$$ respectively.

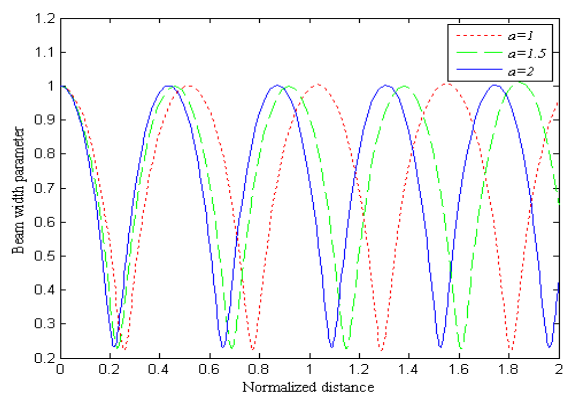
When an intense HGLB propagates in plasma, redistribution of electrons takes place by relativistic-ponderomotive force. Due to the modification of plasma dielectric constant, the refractive index increases at the position of maximum irradiance and the laser gets focused in the plasmas. Equation (20) describes the focusing/defocusing of incident HGLB in collisionless plasma, where the first term on right hand side represent diffraction, which is responsible for the divergence of the laser beam and the second term describe the nonlinear refraction of the beam, which is responsible for the converging behavior of the beam and arises due to the relativistic-ponderomotive force. The relative magnitude of these terms determines the focusing/defocusing behavior of the laser beam in plasma.

We have studied the focusing of HGLB in plasma at the maximum irradiance position, i.e. at $\eta = 0$ for different values of a and n. Figures 1(a) and (b) represent the variation of beam width parameter (f_0) with the dimensionless distance of propagation (ξ) for different orders of HGB (n =1, 2 and 3)) and for different values of intensity parameter (a =1, 1.5 and 2) with fixed value of plasma density ($\omega_{p0}/\omega_0 = 0.3$). The beam suffers oscillatory convergence and divergence during propagation in plasma. It is evident from the figures that the self-focusing of HGLB in plasma is strong for higher values of n and a. This is because the nonlinear refractive term in Eq. (20) is very sensitive to the intensity of the incident laser beam and the order of HGLB. The magnitude of refractive term in Eq. (20) increases with increasing a and n, which leads to strong focusing of HGLB in plasma. In addition, at high intensities of incident laser beam, more electrons contribute to self-focusing. It is important to mention here that the paraxial-ray approximation is valid when $a < 1$. But for HGLB (null intensity at the center), this theory may be valid up to the extent where beam shows strong self-focusing at different order of n. In this case, paraxial-ray approximation is known as modified paraxial-like approach where $r = r_{\max} = r_0\sqrt{2n}$ is the position of the maximum irradiance for the propagating beam [40].

Equation (18) describe the intensity profile of HGLB in plasma in the presence of relativistic and ponderomotive nonlinearities. It depends on the beam width parameters f_0 of the laser beam. The effect of incident laser intensity (a) and the order of the hollow Gaussian beam (n) on the intensity profile of the laser beam in plasma with the normalized distance of propagation is shown in Figures 2(a) and 2(b). One can see from Fig. 2(a) that as the order of the hollow Gaussian beam n increases the intensity of HGLB in plasma is also increases, which implies that with the increase in n the hollow space across the beam also increases. Similarly, it is found that with the increase in the value of a, intensity of HGLB in plasma also increases. This may be due to the strong self-focusing of HGLB in plasma for higher values of n and a. We have used this highly intense self-focused HGLB for the generation of electron plasma wave.



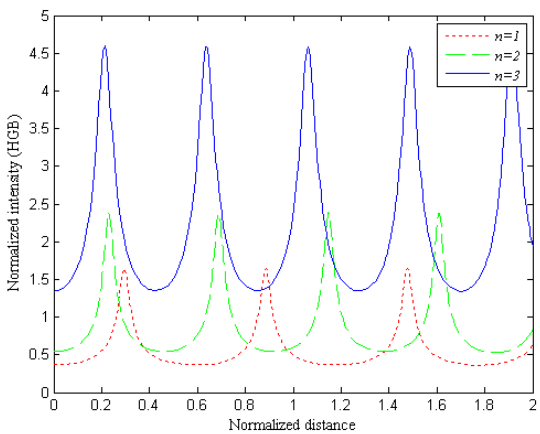
(a)



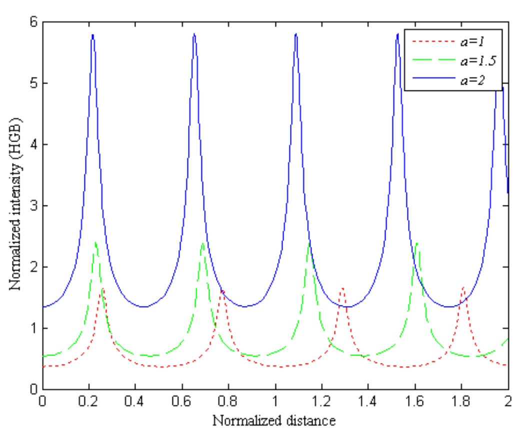
(b)

Figure 1. Variation of beam width parameter (f_0) of hollow Gaussian laser beam with normalized propagation distance (ξ): **(a)** for different orders of HGB (n = 1, 2 and 3) with a = 1.5 and $\omega_{p0} = 0.3\omega_0$ **(b)** for different values of a (= 1, 1.5 and 2) with n = 2 and $\omega_{p0} = 0.3\omega_0$, when both relativistic and ponderomotive nonlinearities are operative.

The electron plasma wave is excited due to nonlinear coupling between HGLB and plasma in the presence of relativistic and ponderomotive nonlinearities. This coupling arises on account of the relativistic change in the electron mass and the modification of the background electron density due to ponderomotive nonlinearity. Equation (30) describes the coupling of EPW with HGLB, while Eq. (28) gives the intensity profile of EPW. The first term in the right hand side of Eq. (30) leads to the divergence of EPW, while the second term is responsible for self-focusing of EPW. It is clear from Eq. (28) the intensity of EPW depends on the focusing of main HGLB and EPW in plasma. The focusing behavior of EPW and scattered wave are the same as HGLB i. e. the extent of focusing is increases with increasing the values of n and a (results not shown here). We have solved Eq. (28) numerically with the help of Eq. (30) to obtain the amplitude of the density perturbation at finite z.



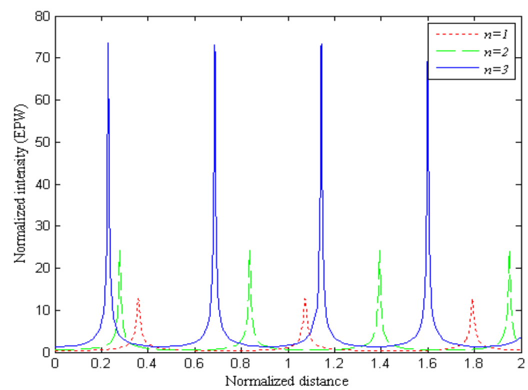
(a)



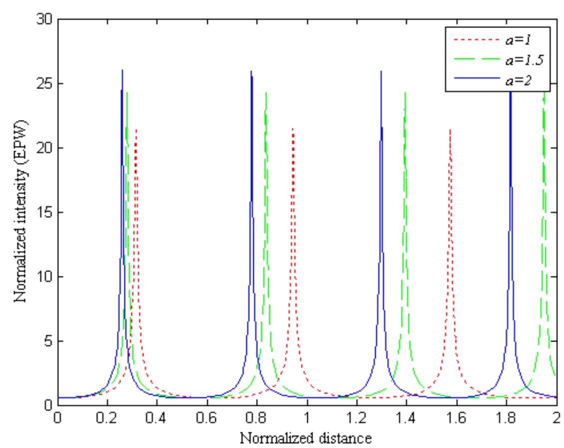
(b)

Figure 2. Variation of normalized laser beam intensity in plasma with normalized distance of propagation (ξ): **(a)** for different orders of HGB (n = 1, 2 and 3) with a = 1.5 and ω_{p0} = 0.3ω_0 **(b)** for different values of a (=1, 1.5 and 2) with n = 2 andω_{p0} = 0.3ω_0, when both relativistic and ponderomotive nonlinearities are operative.

Figures 3 (a) and 3(b) depict the intensity of EPW with normalized distance of propagation for different n and a at the maximum irradiance position i. e. at $\eta = 0$. These figures reflect that the EPW gets excited due to nonlinear coupling with intense laser beam in the presence of the relativistic-ponderomotive nonlinearity. It is evident from the figures that the normalized intensity of EPW increases with increasing the order of HGLB and incident laser intensity respectively. This is due to the fact that intensity of EPW depends on the focusing behavior of HGLB and EPW in plasma.



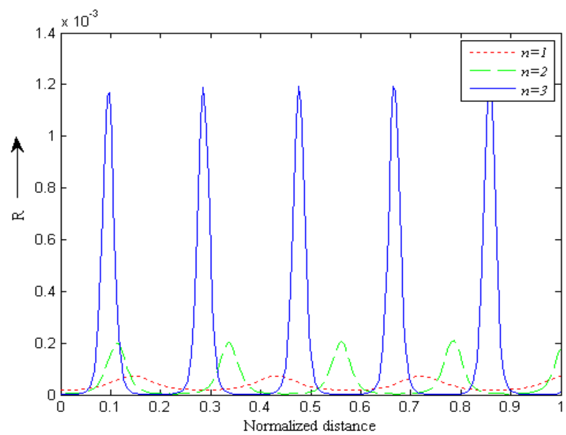
(a)



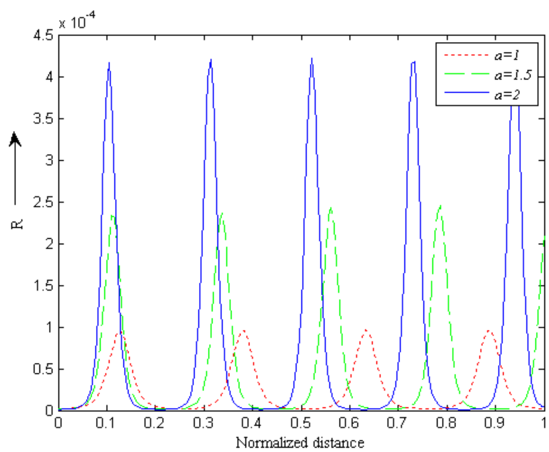
(b)

Figure 3. Variation in normalized intensity of EPW with normalized distance of propagation (ξ): (a) for various orders of HGB (n = 1, 2 and 3) with a = 1.5 andω_{p0} = 0.3ω_0 (b) for different values of a(=1,1.5,2) with n = 2ω_{p0} = 0.3ω_0 and , when both relativistic and ponderomotive nonlinearities are taken into account.

Equations (45) and (49) give the expression for the beam width parameter (f_S) of scattered wave and the backreflectivity (B_R) of SRS against the distance of propagation. It is apparent from the Eq. (49), the back reflectivity of SRS is dependent on the factor $k_{S1}^2 - k_{S0}^2$, intensity of EPW and the focusing of the scattered wave respectively. Equation (49) have been solved numerically to obtain the back reflectivity of SRS with normalized distance for different values of n and a around the maximum irradiance $\eta = 0$. The variation in the back reflectivity of SRS against the normalized distance of propagation for different values of n and a are shown in Figures 4(a) and 4(b) respectively. It is clear that the back reflectivity of SRS increases with increasing n and a. This is due to the fact that the focusing of HGLB, EPW and scattered wave increases with increasing n and a. However, when only ponderomotive or relativistic nonlinearity is operative, the reflectivity is decreases for higher values of n because the focusing of HGLB decreases with increasing order of the beam [43, 44].



(a)



(b)

Figure 4. Variation in back reflectivity (R) of SRS with normalized propagation distance (ξ) when relativistic and ponderomotive nonlinearities are operative: **(a)** for various orders of HGB (n = 1, 2 and 3) with a = 1.5 and $\omega_{p0} = 0.3\omega_0$ **(b)** for different values of a (=1, 1.5 and 2) with n = 2 and $\omega_{p0} = 0.3\omega_0$.

6. Conclusions

In conclusion, we have studied the propagation of an intense laser beam carrying null intensity in center (HGLB) in a collisionless plasma in the presence of relativistic and ponderomotive nonlinearities under WKB and paraxial-ray approximations. The effect of self-focused HGLB on the generation of EPW and back reflectivity of SRS process has been investigated for different values of n and a. The given formulation establishes that the focusing behaviour of HGLB in plasma is significantly enhanced for higher values of n and a. The intensity of the EPW increases for the higher values of n and a due to strong self focusing of HGLB in plasma. It is observed that the backreflectivity of SRS increases at the focused positions for higher order modes of HGLB because the focusing of HGLB and EPW increases for higher order modes. The backreflectivity of SRS also enhance for higher values of pump intensity. The results are useful in the laser induced fusion scheme where the laser beam is superposition of higher order modes.

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References

1. M. Tabak, J. Hammer , M. E. Glinsky, W. L. Kruer, S C. Wilks, J. Woodworth, E. M. Campbell, M. D. Perry, R. J. Mason, Phys. Plasmas, **1994,** 1, 1626-1634. (DOI: https://doi.org/10.1063/1.870664)
2. C. Deutsch, H. Furukawa, K. Mima, M. Murakami, K. Nishihara, Phys. Rev. Lett., **1996**,77, 2483-2486. (DOI: 10.1103/PhysRevLett.77.2483)
3. E. Esarey, C. B. Schroeder, W. P. Leemans, Rev. Mod. Phys., **2009**, 81, 1229-1285. (DOI: https://doi.org/10.1103/RevModPhys.81.1229)
4. R. Bingham, J. T. Mendonca, P. K. Shukla, Plasma Phys. Controlled Fusion, **2004**, 46, R1-R23. (DOI: https://doi.org/10.1088/0741-3335/46/1/R01)
5. D. A. Jaroszynski, R. Bingham, E. Brunetti, B. Ersfeld, J. Gallacher, B. Vander Geer, R. Issac, S. P. Jamison, D. Jones, M. de Loos, A. Lyachev, V. Pavlov, A. Reitsma, Y. Saveliev, G. Vieux, S. M. Wiggins, Phil. Trans. R. Soc. A, **2006**, 364, 689-710. (DOI: 10.1098/rsta.2005.1732)
6. X. Y. Li, J. X. Wang, W. J. Zhu, Y. Ye, J. Li, Y. Yu, Phys. Rev. E, **2011**, 83, 046404. (DOI: https://doi.org/10.1103/PhysRevE.83.046404)
7. P. K. Kaw, G. Schmidt, T. Wilcox, Phys. Fluids, **1973**, 16, 1522-1525. (DOI: https://doi.org/10.1063/1.1694552)
8. G. Z. Sun, E. Ott, Y. C. Lee, P. Gujdar, Phys. Fluids, **1987**, 30, 526-532. (DOI: https://doi.org/10.1063/1.866349)
9. W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison- Wesley Publishing Company: California, 1988)
10. N. A. Krall, A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill: New York, 1973)
11. C. S. Liu, V. K. Tripathi, *Interaction of Electromagnetic*

Waves with Electron Beams and Plasmas (World Scientific: Singapore, 1994)
12. W. B. Mori, C. Joshi, J. M. Dawson, D. W. Forslund, J. M. Kindel, Phys. Rev. Lett., **1988**, 60, 1298-1301.(DOI: https://doi.org/10.1103/PhysRevLett.60.1298)
13. P. M. Lushnikov and H. A. Rose, Plasma Phys. Controlled Fusion, **2006**, 48, 1501-1513. (DOI: https://doi.org/10.1088/0741-3335/48/10/004)
14. A. J. Palmer, Phys Fluids, **1971**, 14, 2714-2718. (DOI: https://doi.org/10.1063/1.1693396)
15. A. Salcedo, R. J. Focia, A. K. Ram, A. Bers, Nucl. Fusion, **2003**, 43 1759-1770. (DOI: https://doi.org/10.1088/0029-5515/43/12/020)
16. C. Labaune, J. Fuchs, S. Depierreux, H. A. Baldis, D. Pesme, J. Myatt, S. Hüller, V. T. Tikhonchuk, G. Laval, Comptes Rendus de l'Académie des Sciences - Series IV - Physics, **2000**, 1, 727-735. (DOI: https://doi.org/10.1016/S1296-2147(00)01078-7)
17. C. Labaunea, Teller Medal Lecture EPJ Web of Conferences, **2013**, 59, 01012. (DOI: 10.1051/epjconf/20135901012)
18. H. A. Baldis, Plasma Phys., 1983, 25, 1297-1310. (DOI: https://doi.org/10.1088/0032-1028/25/12/302)
19. C. S. Liu and V. K. Tripathi, Phys. Fluids, **1986**, 29, 4188-4191. (DOI: https://doi.org/10.1063/1.865710)
20. H. C. Barr, T. J. M. Boyd, G. A. Coutts, Phys. Rev. Lett., **1986**, 56, 2256-2259. (DOI: https://doi.org/10.1103/PhysRevLett.56.2256)
21. R. W. Short, W. Seka, R. Bahr, Physics of Fluids, **1987**, 30, 3245-3251. (DOI: https://doi.org/10.1063/1.866499)
22. T. Afsar-rad, S. E. Coe, O. Willi, M. Desselberger, Phys. Plasmas, **1992**, 4, 1301 -1322. (DOI: https://doi.org/10.1063/1.860086)
23. E. S. Dodd and D. Umstadter, Phys. Plasmas, **2001**, 8, 3531-3534. (DOI: 10.1063/1.1382820)
24. T. Y. Hui, H. Shen- Sheng, Z. Chang- Xue, W. Yan- Qing, C. Jing, Z. Fang-Chuan, Z. Yu-Zhe and X. Zhi-Zhan, Chinese Phys., **2002**, 11, 50-53. (DOI: 1009-1963/2002/11(01)/0050-04**)**
25. R. K. Kirkwood, R. L. Berger, C. G. R. Geddes, J. D. Moody, B. J. Macgowan, S. H. Glenzer, K. G. Estabrook, C. Decker and O. L. Landen, Phys. Plasmas, **2003**, 10, 2948-2955. (DOI: https://doi.org/10.1063/1.1580814)
26. Y. S. Kalmykov and G. Shvets, Phys. Plasmas, **2004**, 11, 4686-4694. (DOI: https://doi.org/10.1063/1.1778743)
27. P. Jha, G. Raj and A. K. Upadhyaya, IEEE Trans. Plasma Sci., 2006, 34, 922 -926. (DOI: 10.1109/TPS.2006.878144)
28. J. L. Kline, D. S. Montgomery, C. Rousseaux, S. D. Baton, V. Tassin, R. A. Hardin, K. A. Flippo, R. P. Johnson, T. Shimada, L. Yin, B. J. Albright, H. A. Rose and F. Amiranoff, Laser Part. Beams, 2009, 27, 185-190. (DOI: https://doi.org/10.1017/S0263034609000251)
29. A. Paknezhad, and D. Dorranian, Laser Part. Beams, **2011**, 29, 373-380. **(**DOI: 10.1017/S0263034611000474)
30. G. Purohit, P. Sharma, R. P. Sharma, J. Plasma Phys., **2012**, 78, 55-63. (DOI: https://doi.org/10.1017/S0022377811000419)
31. S. C. Wilks, W. L. Kruer, E. A. Williams, P. Amendt**,** D. C. Eder, Phys. Plasmas, **1995**, 2, 274-279. (DOI: https://doi.org/10.1063/1.871097)
32. H. A. Salih, S. T. Mahmoud, R. P. Sharma, Phys. Plasmas, **2005**, 12, 042302. (DOI: https://doi.org/10.1063/1.1856480)
33. D. N. Gupta, S. Yadav, V. K. Tripathi, H. Suk, Phys. Scr., **2006**, 73, 284 -287. (DOI: https://doi.org/10.1088/0031-8949/73/3/008)
34. P. Sharma and R. P. Sharma, Phys. Plasmas, **2009**, 16, 032301. (DOI: https://doi.org/10.1063/1.3077670)
35. A. Singh and K. Walia, Optik, **2013**, 124, 6074-6080. (DOI: https://doi.org/10.1016/j.ijleo.2013.07.005)
36. A. Singh and K. Walia, Contrib. Plasma Phys., **2011**, 51**,** 788-797. (DOI: 10.1002/ctpp.201000042)
37. P. Rawat, R. Gauniyal, G. Purohit, Phys. Plasmas, **2014**, 21**,** 062109. (DOI: https://doi.org/10.1063/1.4883221)
38. Y. Cai, X. Lu, Q. Lin, Opt. Lett., **2003**, 28, 1084-1086. (DOI: https://doi.org/10.1364/OL.28.001084)
39. Y. Cai and Q. Lin, J. Opt. Soc. Am. A, **2004**, 21, 1058-1065. (DOI: https://doi.org/10.1364/JOSAA.21.001058)
40. M. S. Sodha, S. K. Misra, S. Misra, Laser Part. Beams, **2009**, 27 , 57-68. (DOI: https://doi.org/10.1017/S026303460900008)
41. A. Sharma, S. Misra, S. K. Mishra, I. Kourakis**,** Phys. Rev. E, **2013**, 87, 063111. (DOI :https://doi.org/10.1103/PhysRevE.87.063111)
42. N. P. Chaitanya, M. V. Jabir, J. Banerji, G. K. Samanta, Sci. Rep., **2016**, 6, 32464. (DOI: 10.1038/srep32464)
43. R. K. Singh and R. P. Sharma, Laser Part. Beams, **2013**, 31, 387-394. (DOI: https://doi.org/10.1017/S0263034613000384)
44. P. Sharma, Laser Part. Beams, **2015**, 33, 489-498. (DOI: https://doi.org/10.1017/S0263034615000488)
45. S. A. Akhmanov, A. P. Sukhorokov, R. V. Kokhlov, Sov. Phys. Uspekhi, 1968, 10, 609-636. (DOI: https://doi.org/10.1070/PU1968v010n05ABEH005849)
46. M. S. Sodha, A. K. Ghatak, V. K. Tripathi, Prog. in Optics, **1976**, 13, 169-265. (DOI: https://doi.org/10.1016/S0079-6638(08)70021-0)
47. H. S. Brandi, C. Manus, G. Mainfray, Phys. Fluids B, **1993**, 5, 3539-3550. (DOI: https://doi.org/10.1063/1.860828)
48. A. Kumar, M. K. Gupta, R. P. Sharma, Laser Part. Beams, **2006**, 24, 403-409. (DOI: https://doi.org/10.1017/S0263034606060563)
49. S. Misra and S. K. Mishra, Prog. in Electmagnetic Research B, **2009**, 16, 291-309. (DOI: 10.2528/PIERB09061705)
50. J. F. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, M. N. Rosenbluth, Phys. Fluids, **1974**, 17, 778-785. (DOI: https://doi.org/10.1063/1.1694789)